ACKNOWLEDGMENT

The authors wish to thank F. Kato of the University of Tokyo for his helpful suggestions concerning the application of the Newton–Raphson method.

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Short Papers

Propagation in Circular Waveguide Loaded with an Azimuthally Magnetized Ferrite Tube

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Abstract—**A new computer method for the study of radially inhomogeneous guiding structures presenting circular symmetry is utilized to determine propagation properties of a ferrite loaded guide. The nonreciprocal characteristics obtained can be used to design latching rotators and differential phase shifters for polarization orthogonality restoration in high-frequency (above 10 GHz) communication systems with frequency reuse.**

I. INTRODUCTION

A circular waveguide loaded with one or several tubes of azimuthally magnetized ferrite supports nonreciprocal modes of propagation, which may be used to realize rotators and differential phase shifters. This possibility was briefly outlined by Fox *et al.* [1] and by Clarricoats [2].

Until now, however, the studies devoted to this guiding structure only considered TE_{0n} modes, for which an exact analytical solution is available in terms of hypergeometric functions [3]–[6]. A perturbation method also developed for this structure was applied only to study TE_{0n} mode propagation [7]. It must be noted, however, that the dominant mode of the empty circular waveguide is the TE_{11} mode, which is not part of the TE_{0n} mode subset; the mode hierarchy remains basically the same for low to medium loading of the waveguide, i.e., conditions most suitable for device operation. Only when the waveguide is more heavily loaded with dielectric inserts does the mode inversion described by Tsandoulas and Ince [8], [9] take place: The TE_{01} mode then becomes the dominant mode. By using a circular waveguide with dielectric loading, results available for TE_{01} propagation could thus be utilized in the design of devices [10]. Partial studies only are available in the technical literature, due to the lack of a mathematical method capable of solving the field problem for hybrid modes.

In practical device development, however, a more complete knowledge of the mode structure is necessary; the propagation properties of the dominant mode must be known in order to design the device, while the cutoff frequency of the first higher order mode limits the frequency range suitable for operation.

A computation technique, based on the application of a one-dimensional finite-difference approach, was recently developed by the authors [11]. This new approach yields the propagation constant and the field pattern for any mode in a radially inhomogeneous cylindrical structure presenting circular symmetry, of which the ferrite loaded circular waveguide considered here is one example.

The presence of azimuthally magnetized ferrite in the circular waveguide produces a difference between the propagation characteristics of the first two normal modes propagating in the structure, which are respectively right hand and left hand circularly polarized (in nongyrotropic circularly symmetrical structures, these two modes are spatially degenerate). When a linearly polarized wave travels through a section of loaded guide, its plane of polarization is rotated in the same manner as it is in longitudinally magnetized Faraday rotators [12]. The addition of quarter-wave plates at both ends allows one to realize a polarization-dependent phase shifter; the principle of operation is the same as the one used by Boyd [13], with the following differences:

1) The center section makes use of an azimuthally magnetized ferrite tube instead of a Faraday rotator, which requires an external magnetic circuit.

2) Both device ends are connected to circular waveguide.

Manuscript received April 2, 1976; revised July 19, 1976. This work was supported by the Swiss National Science Foundation under Grant 2, 346, 0, 75.
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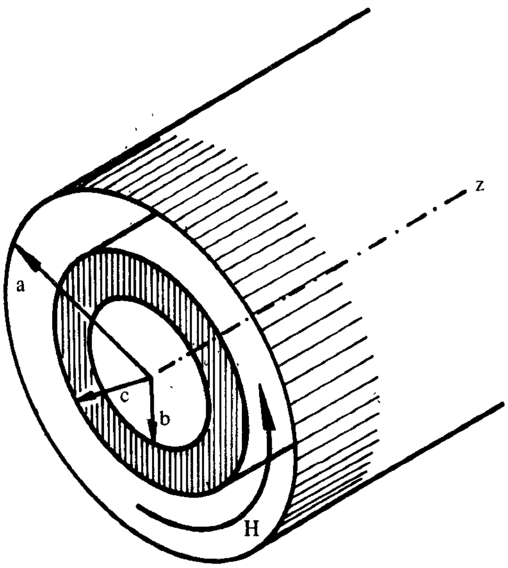

Fig. 1. Circular waveguide containing one azimuthally magnetized ferrite tube.

The two functions of polarization rotation and polarization-dependent phase shift cannot be realized in rectangular waveguide (where the dominant mode is not degenerate) nor with the TE_{01} mode in circular waveguide (which does not possess one plane of polarization).

Polarization rotators and polarization-dependent phase shifters in circular guide present a great interest for microwave propagation links above 10 GHz presently being developed. Frequency reuse techniques, by means of which channel capacity can be doubled using simultaneous transmission over the two polarizations, are affected by atmospheric disturbances, which reduce the cross-polar discrimination. A scheme to restore polarization orthogonality was proposed by Chu [14]: It requires phase-dependent phase shifters and attenuators, as well as rotating joints. A realization using mechanical components driven by servo-motors was recently proposed [15]. Ferrite devices can fulfill the same functions without moving parts: Geometrical rotation is replaced by a ferrite rotator controlled by an electrical current. As they do not possess any moving part, ferrite devices are less sensitive to wear and require less maintenance than their mechanical counterparts, while providing faster response times. Last but not least, latching devices using the geometry considered in the present publication are best suited to digital control by a computer and do not require any holding current for operation.

II. Description of the Model Considered

The method developed in [11] provides an accurate means of resolution for wave propagation and field pattern in circular waveguides loaded with one or several tubes of magnetized ferrite and of lossless or lossy dielectric. For the sake of clarity, the present article will deal only with one single tube of ferrite azimuthally magnetized to remanence, as sketched in Fig. 1. The waveguide radius is denoted by a, the ferrite extending between b (inside radius) and c (outside radius).

The tensor permeability of the ferrite magnetized to remanence along the e_ϕ direction is given by

$$\bar{\mu} = \mu_0 \begin{pmatrix} \mu_r & 0 & jK \\ 0 & 1 & 0 \\ -jK & 0 & \mu_r \end{pmatrix} \qquad (1)$$

with the value of the tensor components given by [16]

$$\mu_r = 1 - j\frac{\pi M_s \Delta H_e}{(f/\gamma_g)^2 + (\Delta H_e/2)^2}(1 + R^2)$$
$$\simeq 1 - j\frac{\gamma_g{}^2 \pi M_s \Delta H_e}{f^2}(1 + R^2) \qquad (2)$$

$$K = -\frac{4\pi M_s (f/\gamma_g) R}{(f/\gamma_g)^2 + (\Delta H_e/2)^2} \simeq -\frac{\gamma_g 4\pi M_s R}{f} \qquad (3)$$

where $4\pi M_s$ is the saturation magnetization, $R = M_r/M_s$ is the remanence ratio, f is the frequency of operation, γ_g is the gyromagnetic ratio, and ΔH_e is the effective linewidth at remanence (which can be much smaller than the resonance linewidth for polycrystalline ferrites). It is significant to note that, at remanence, only the diagonal terms of the tensor contribute losses, while differential phase shift results from the nondiagonal terms only. The approximation [right-hand term of (2) and (3)] is valid when $\gamma_g \Delta H_e \ll f$.

III. Brief Description of the Computer Method

Since the technique used for the computation is described in detail in [11], only a brief description of the principle will be given here. The circular cross section of the waveguide is divided into a number of thin concentric rings, magnetic and dielectric material properties within each ring being uniform. The radial variation of the components along ϕ and z of the electromagnetic field is expressed in terms of the components themselves, utilizing Maxwell's equations for steady-state sinusoidal operation and assuming complex field dependences of the form

$$\exp(j\omega t + jm\phi - \gamma z) \qquad (4)$$

where ω is the radial frequency, m is the azimuthal mode number, and γ is the propagation constant. Transfer matrices for each ring are then determined by numerical integration (Runge–Kutta): The four components at the outside of each ring are linked to those at the inside by a 4×4 matrix. Since these components are continuous at interfaces between rings (no surface currents are assumed), the total transfer matrix is obtained by successive multiplication of the transfer matrices of each ring. Applying the boundary conditions at the waveguide center and wall provides the dispersion relation, which is solved numerically for the propagation constant γ. The field distribution (six components) can then be calculated. The same computation process is applied to both directions of propagation for any mode. The number of concentric rings is determined to give sufficient accuracy while keeping decent computation times. A comparison with the exact mathematical resolution for the TE_{01} mode in the twin-toroid ferrite geometry of [10] showed an agreement of the order of 0.1 percent.

IV. Computer Results

Computations were carried out to determine the effect of frequency, geometry, and material properties for the dominant hybrid mode (HE_{11}) of a waveguide loaded with tubes of azimuthally magnetized ferrite. The effect of waveguide loading is shown in Fig. 2, considering, respectively, a tube extending from the center (thin hole with $b/a = 0.05$) to a varying outer radius c [Fig. 2(a)], and a tube extending from the outside wall to a varying inner radius b [Fig. 2(b)]. In both cases are represented the phase shifts per unit length β for forward waves of the two circularly polarized modes ($m = +1$ and $m = -1$) for a 1-cm-radius waveguide at a frequency of 10 GHz with ferrite

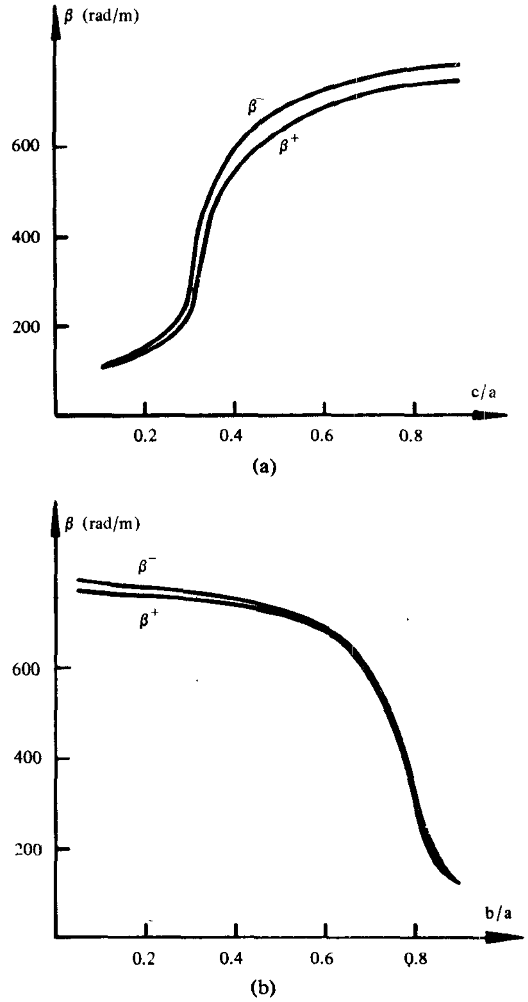

Fig. 2. Propagation constant for the dominant quasi-TE_{11} mode in the loaded structure (both directions of propagation). (a) With a ferrite rod at the waveguide center as a function of filling factor c/a (with a small hole $b/a = 0.05$). (b) With a ferrite tube next to the outer waveguide wall ($c = a$) as a function of filling factor b/a. In both cases $f = 10$ GHz, $a = 1$ cm, $K = 0.2$, and $\varepsilon_f = 15$.

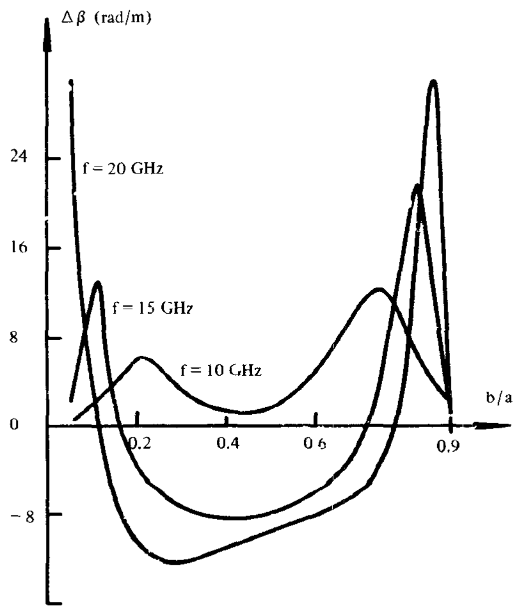

Fig. 3. Differential phase shift for an azimuthally magnetized ferrite tube having a constant thickness $c - b = 0.1a$ as a function of location b within the guide, for various frequencies (with $K = 2 \times 10^{+9}/f$).

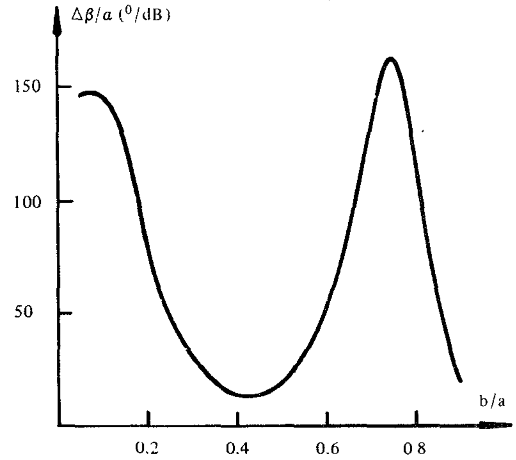

Fig. 4. Quality factor as a function of tube location as per Fig. 3 at 10 GHz. Attenuation is due to copper losses in outer conductor ($\rho = 17$ nΩm), dielectric losses in ferrite ($\tan \delta = 0.07$ percent) and magnetic losses ($\mu_r = 1 - j0.0088$).

relative permittivity $\varepsilon_f = 15$ and off-diagonal relative permeability $K = 0.2$. The angle of rotation per unit length of the plane of polarization is given by

$$\theta = (\beta_+ - \beta_-)/2 = \Delta\beta/2. \quad (5)$$

Fig. 3 shows the differential phase shift per unit length $\Delta\beta$ provided by a ferrite tube of thickness $0.1a$ for several frequencies as a function of radial position. Two peaks of the differential phase shift appear: one of them when the tube is close to the center, and another one near the outer boundary of the waveguide. Between these two peaks, the phase shift varies considerably with frequency, actually changing sign. (This effect may be of interest in devices other than usual latching phase shifters, where a constant differential phase shift is usually preferred.)

Losses must also be taken into account when selecting the structure best suited for device implementation. A quality factor, defined by the differential phase shift to attenuation ratio, is presented in Fig. 4 at 10 GHz. The attenuation is due to copper conduction losses in the outer wall, dielectric and magnetic losses within the ferrite. The best performance is obtained here at 10 GHz with $b/a = 0.735$.

The effect of a variation of magnetic permeability was investigated for the two maxima positions of Fig. 4. In both instances the differential phase shift per unit length $\Delta\beta$ was found to be nearly proportional to the off-diagonal tensor component K. The frequency bandwidth of the structure was also determined: It was found to be rather narrow, limited to about 13 percent by the onset of higher order modes for the condition of best performance determined in Fig. 4. If broad-band operation is desired, steps should be taken to suppress the excitation of higher order modes.

For the polarization compensation described earlier, the phase shifters should provide a relatively constant phase shift over a specified frequency band. Fig. 5 shows the frequency dependence over the 9–11-GHz band for ferrite tubes of slightly different radii. If operation over a wide range of frequencies is required, some broad-banding technique should be utilized to flatten the phase shift curves, for instance dielectric loading, or use of rings having different diameters.

All calculations were carried out on the CDC Cyber 7326 of the Computer Center of the École Polytechnique Fédérale. Programs were written in Fortran IV.

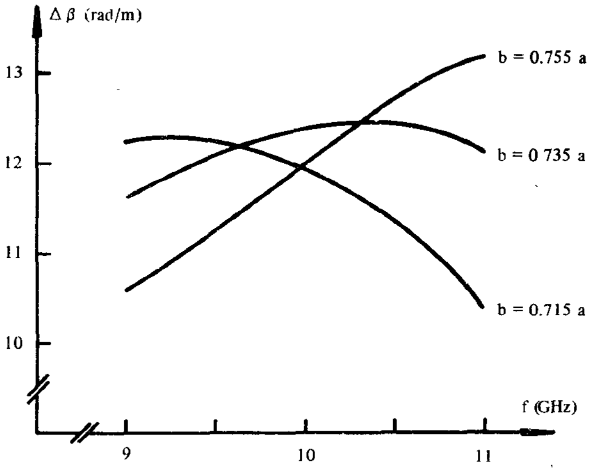

Fig. 5. Differential phase shift as a function of frequency for $a = 1$ cm, $c - b = 1$ mm, $\varepsilon_f = 15$, $K = 2 \times 10^{+9}/f$

V. Conclusion

A general technique to study propagation in loaded circular cylindrical guiding structures was developed and applied to a cylindrical waveguide containing a coaxial tube of ferrite azimuthally magnetized to remanence (no accurate solution was previously available for this structure). This geometry can be used to realize latching rotators and phase-dependent phase shifters for cross-polarization compensation in high-frequency telecommunications.

Computer results were presented for one particular structure showing the influence of geometric parameters, material properties, and frequency. Since a large number of parameters are involved, it is not possible to present a more general description. It is hoped that the results presented will give a general idea of the behavior of the structure considered. The presence of a thin latching conductor at the waveguide center produces a negligible effect on the propagation constant of the HE_{11} modes. On the other hand, precautions must be taken not to launch coaxial-line modes over the loaded section. The side connections of the conductor are equivalent to shunt capacitors for one of the linearly polarized modes. Their effect can, however, be compensated over a broad frequency band by the addition of a series inductance at the same location [17]. Device optimization should take into account specified frequency bands and parameters of ferrite materials actually available in tube form. The authors would be pleased to provide, on a complimentary basis, copies of the computer listing to readers interested in pursuing further the study presented here.

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Behavior of the Magnetostatic Wave in a Periodically Corrugated YIG Slab

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***Abstract*—An analysis for the propagation characteristics of the magnetostatic wave in a YIG slab having periodically corrugated surfaces is presented. The Brillouin diagrams, close to the intersection point (ω,K) of $m = 0$ and $m = -1$ space harmonics, are obtained for different slab thicknesses, and the nonexistence of leaky wave modes has been established. Some discussions concerning the internal dc magnetic field and the propagation loss are also presented.**

I. Introduction

The periodic structures in dielectric media have long been a center of attraction of many researchers, particularly for their usefulness in many devices like filters, surface wave antennas, and distributed feedback (DFB) amplifiers in microwave and optical integrated circuits [1]–[4]. Recently, one of the present authors treated the case of propagation characteristics of magnetostatic waves in periodically magnetized ferrites where the internal dc magnetic field was modulated by providing additional magnets which were placed periodically around the YIG sample [5]. Also Elachi, in his paper [6], has studied the propagation of magnetic waves in an infinite periodic medium where he considered the dielectric constant of the medium to vary sinusoidally in space and treated the case of DFB-type magnetic wave oscillators. However, the effect of periodicity of the dielectric constant is rather small, because of the fact that a magnetic wave can propagate with no electric field components [7].

The present short paper investigates the propagation of the magnetostatic wave through a YIG slab having a periodically

Manuscript received April 13, 1976; revised August 30, 1976.
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